A list of statements/theorems that you should be able to prove.

1. If $f : X \to Y$ is a map and $A_1, A_2 \subset X$, $B_1, B_2 \subset Y$ are subsets, then
$$f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$$
$$f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$$
$$f(A_1 \cup A_2) = f(A_1) \cup f(A_2).$$

2. If A and B are countable sets, then $A \times B$ is countable.

3. The set of rationals is countable.

4. The set of infinite sequences whose elements are all 0 or 1 is uncountable.

5. Suppose that M, N are subsets of a metric space X. The closure operation satisfies the following properties:
   (a) If $M \subset N$ then $[M] \subset [N]$.
   (b) $[[M]] = [M]$.
   (c) $[M \cup N] = [M] \cup [N]$.
   (d) $[\emptyset] = \emptyset$.

6. Closed and open subsets of a metric space satisfy the following properties:
   (a) The intersection of an arbitrary collection of closed sets is closed.
   (b) The union of finitely many closed sets is closed.
   (c) The intersection of finitely many open sets is open.
   (d) The union of an arbitrary collection of open sets is open.

7. A subset $M \subset R$ in a metric space R is open if and only if the complement $R \setminus M$ is closed.

8. Every convergent sequence in a metric space is a Cauchy sequence.

9. (Nested sphere theorem) Let R be a complete metric space, and $\overline{B}_{r_k}(x_k)$ a sequence of closed balls in R such that
$$\overline{B}_{r_1}(x_1) \supset \overline{B}_{r_2}(x_2) \supset \dots,$$
and $r_k \to 0$ as $k \to \infty$. Then the intersection $\bigcap_{k \geq 1} \overline{B}_{r_k}(x_k)$ is non-empty.

10. (Baire's theorem) Let R be a complete metric space, and suppose that
$$R = \bigcup_{k=1}^{\infty} A_k$$
for a sequence of subsets $A_k \subset R$. Then it is not possible that all of the A_k are nowhere dense.

11. Let R be a complete metric space, and $F : R \to R$ a map such that for some $\alpha < 1$ we have

$$\rho(F(x), F(y)) \leq \alpha\rho(x, y)$$

for all $x, y \in R$. Then there exists a unique point $x \in R$ for which $F(x) = x$.

12. A map $f : X \to Y$ between topological spaces is continuous, if and only if $f^{-1}(U)$ is open for every open set $U \subset Y$.

13. The interval $[0, 1]$ is connected.

14. If a topological space X is path connected, then it is connected.

15. Let $f : X \to Y$ be continuous, and suppose that X is connected. Then $f(X)$ is connected.

16. A topological space X is compact if and only if every centered system of closed sets in X has non-empty intersection.

17. If X is compact and $F \subset X$ is closed, then F is compact.

18. Suppose that X is a Hausdorff space, and $K \subset X$ is compact. Then K is closed.

19. Let X be compact, and $f : X \to Y$ continuous. Then $f(X)$ is compact.

20. Any sequence in a compact metric space has a convergent subsequence.

21. A metric space R is compact if and only if it is complete and totally bounded.

22. A subset $M \subset C_{[0,1]}$, with the distance $\rho(f, g) = \sup_{x\in[0,1]} |f(x) - g(x)|$ is totally bounded, if and only if it is uniformly bounded and equicontinuous.

23. If $F : X \to \mathbf{R}$ is continuous, and the metric space X is compact, then F is uniformly continuous.

24. If $F : X \to \mathbf{R}$ is continuous and X is compact, then $F(X)$ is bounded, and F achieves its infimum and supremum.

25. If $F : X \to \mathbf{R}$ is lower semicontinuous and X is compact, then $F(X)$ is bounded from below, and F achieves its infimum.

26. If V is a normed linear space and $W \subset V$ is a closed subspace, then the quotient V/W is also a normed linear space with the norm

$$\|[x]\|_{V/W} = \inf\{\|x - y\|_V \,;\, y \in W\}.$$

27. If a linear functional $f : V \to \mathbf{R}$ on a normed linear space is continuous at a point, then it is continuous everywhere.

28. A linear functional $f : V \to \mathbf{R}$ on a normed linear space is continuous if and only if it is bounded.

29. If V is a normed linear space, then its conjugate space V^* is complete, i.e. it is a Banach space.

30. If V is a normed linear space and $x \in V$ a non-zero element, then there is a continuous linear functional $f \in V^*$ such that $\|f\| = 1$ and $f(x) = \|x\|$.